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What measurable zero point fluctuations can(not) tell us about dark energy.

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We show that laboratory experiments cannot measure the absolute value of dark energy. All known experiments rely on electromagnetic interactions. They are thus insensitive to particles and fields that interact only weakly with ordinary matter. In addition, Josephson junction experiments only measure differences in vacuum energy similar to Casimir force measurements. Gravity, however, couples to the *absolute* value. Finally we note that Casimir force measurements have tested zero point fluctuations up to energies of $\sim 10\,\mathrm{eV}$, well above the dark energy scale of $\sim 0.01\,\mathrm{eV}$. Hence, the proposed cut-off in the fluctuation spectrum is ruled out experimentally.

I. INTRODUCTION

Recently, a measurement of dark energy in the lab using noise in Josephson junctions has been proposed [1]. While it would be wonderful to see dark energy in the lab, several groups argued that this will unfortunately *not* be possible [2, 3, 4, 5]. In the course of preparing this article, two groups [4, 5] published results similar to the following arguments. We nevertheless would like to present our view of the topic, hoping that it might help to clarify some aspects and settle the ongoing discussion.

Our reasoning is split in three parts: first, we argue that all devised measurements of zero point fluctuations are based on the coupling to the electromagnetic field (Sect. II). They would hence not measure all zero point fluctuations, but only those of particles coupled electromagnetically. Secondly, we show in Sect. III that the fluctuation-dissipation theorem is invariant under a shift in vacuum energy. This has already been observed by [4]. Thirdly, in Sect. IV we discuss experimental results for the Casimir force. Even if such experiments were to measure absolute energies of zero point fluctuations (which they can't), the cut off in frequency needed to describe dark energy is not seen there. This argument has been put forward earlier [2] and presented in more detail recently [5].

In a nutshell, our conclusion is: Although experiments studying Josephson junctions and the Casimir effect might be interpreted as measuring zero point fluctuations, they cannot yield the absolute amount of dark energy.

II. MEASUREMENTS NEED COUPLINGS TO THE ELECTRIC FIELD

We are going to show that both Josephson junction type and Casimir effect experiments are only sensitive to particles that couple to the $U(1)$ of the electromagnetic field. To begin, consider the Casimir force between two ideal parallel plates (see, e.g. [7]),

$$F(a) = -\frac{\partial E_0^{\mathrm{ren}}(a)}{\partial a}, \quad E_0^{\mathrm{ren}}(a) = -n_{\mathrm{p}} \frac{c\hbar\pi^2}{720 a^3}, \qquad (1)$$

where $n_{\mathrm{p}} = 2$ is the number of photon polarizations and a is the distance between plates. The Casimir force (1) does not explicitly depend on the electromagnetic coupling. Why then do only the fluctuations of the electromagnetic field $n_{\mathrm{p}} = 2$ contribute? In other words, why is n_{p} not replaced by the total number of degrees of freedom?

One might be tempted to attribute this to the suppression of contributions of massive particles to the Casimir force $\sim \exp(-2mca/\hbar)$. Yet, modern experiments (cf. also Sect. IV) probe scales $\frac{2\pi}{a} \sim 10\,\mathrm{eV}$ where neutrinos are effectively massless. No additional contributions from neutrinos have been found (as expected). The solution is the very weak interaction of neutrinos with the plates. To neutrinos, the plates are translucent and provide no boundary condition. Photons, on the other hand, lead to a rearrangement of charges in the ideally conducting plates such that the electric field vanishes. The charges enforce the boundary conditions as a consequence of their coupling to the electromagnetic field (the limit of ideally conducting plates corresponds to an infinitely large coupling caused by the infinite number of charges in the plates). Accordingly, photons contribute to the Casimir force as specified in Eq. (1). From Eq. (1), we also see that the Casimir force measures the derivative of the zero point fluctuations and is thus insensitive to an overall shift Λ in vacuum energy.

Turning to Josephson junctions, one measures the quantum mechanical noise in voltage due to the noise of the electric current. From this, the special role of electromagnetically interacting particles is rather obvious. In particular, Josephson junctions do not measure fluctuations of exotic weakly coupled particles or even neutrinos.

III. A CLOSER LOOK AT THE NOISE IN JOSEPHSON JUNCTIONS

The basis for the interpretation of the noise in Josephson junctions as a measurement of vacuum fluctuations is the so called fluctuation dissipation theorem,

$$\langle V^2 \rangle = \frac{2}{\pi} \int_0^\infty R(\omega) \left(\frac{1}{2}\hbar\omega + \frac{\hbar\omega}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1} \right). \qquad (2)$$

Here V is a "force" and $R(\omega)$ the "resistance". The term

$$\frac{1}{2}\hbar\omega + \frac{\hbar\omega}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1} \tag{3}$$

closely resembles the vacuum energy of an harmonic oscillator. Although this certainly is an effect of zero point fluctuations, it does not depend on the absolute value of the vacuum energy: shifting the Hamiltonian by an arbitrary constant Λ (thereby changing the "dark energy" by this constant) Eqs. (2) and (3) remain unaffected. The constant Λ will not appear in the fluctuation-dissipation theorem.

To see this more clearly let us briefly review the derivation of Eq. (2) as given in [8]. Starting point is a Hamiltonian of the form

$$H = H_0(\cdots q_k \cdots p_k \cdots) + VQ(\cdots q_k \cdots p_k \cdots). \tag{4}$$

H_0 is the unperturbed Hamiltonian and V is a function of time that measures the magnitude of the perturbation while Q depends only on the coordinates and momenta. In time dependent (quantum mechanical) perturbation theory one can now calculate the power absorbed by the system in an energy state E_n when an outside force of the form $V = V_0 \sin(\omega t)$ is applied,

$$\begin{aligned} P(E_n) = \frac{1}{2}\pi V_0^2 \omega \big[&|\langle E_n + \hbar\omega | Q | E_n\rangle|^2 \rho(E_n + \hbar\omega) \\ &- |\langle E_n - \hbar\omega | Q | E_n\rangle|^2 \rho(E_n - \hbar\omega)\big], \end{aligned} \tag{5}$$

where $\rho(E)$ is the density of states with energy E. Weighting with a Boltzmann factor $f(E_n) = \exp(-E/kT)/\mathcal{Z}$ and summing over all energy states one obtains the total power absorbed by the system,

$$\begin{aligned} P_{tot} = \frac{1}{2}\pi V_0^2 \omega \sum_n \big[&|\langle E_n + \hbar\omega | Q | E_n\rangle|^2 \rho(E_n + \hbar\omega) \\ &- |\langle E_n - \hbar\omega | Q | E_n\rangle|^2 \rho(E_n - \hbar\omega)\big] f(E_n). \end{aligned} \tag{6}$$

Let us now shift the Hamiltonian by a constant Λ (we label quantities in the "new" system with a tilde). For the matrix elements, the shift Λ amounts to relabeling $E_n \to \tilde{E}_n = E_n + \Lambda$, i.e. the n-th state now has energy $E_n + \Lambda$ but still the same wave function. Therefore, the matrix element remains the same. Likewise, the argument of the density of states is shifted by a constant Λ, $\rho(E_n) \to \tilde{\rho}(\tilde{E}_n) = \tilde{\rho}(E_n + \Lambda) = \rho(E_n)$.

The only place where the energy enters explicitly is the Boltzmann factor f. However, requiring proper normalization,

$$\sum_n f(E_n)\rho(E_n) = 1 = \sum_n f(\tilde{E}_n)\tilde{\rho}(\tilde{E}_n), \tag{7}$$

one easily finds using $\rho(E_n) = \tilde{\rho}(\tilde{E}_n)$ that

$$f(E_n) = \tilde{f}(\tilde{E}_n). \tag{8}$$

All in all, P_{tot} is unaffected by the shift Λ in energy. Using the definition of impedance,

$$V = Z(\omega)\dot{Q} \tag{9}$$

one has for the average dissipated power

$$P_{tot} = \frac{1}{2}V_0^2 \frac{R(\omega)}{|Z(\omega)|^2}, \quad R(\omega) = Re(Z(\omega)). \tag{10}$$

Using $\sum_n(\ ) \to \int_{-\infty}^{\infty}(\ )\rho(E)dE$ we can replace the sum over states in Eq. (6) with an integral and compare to Eq. (10),

$$\begin{aligned} \frac{R(\omega)}{|Z(\omega)|^2} = \pi\omega \int_{-\infty}^{\infty} dE\, \rho(E) f(E) & \\ \times \big[|\langle E + \hbar\omega | Q | E\rangle|^2 \rho(E + \hbar\omega) & \\ - |\langle E - \hbar\omega | Q | E\rangle|^2 \rho(E - \hbar\omega)\big]. & \end{aligned} \tag{11}$$

Please note that although the integral goes from $-\infty$ to ∞ it effectively has a finite lower limit since the density of states vanishes below the ground state energy of the system.

The next important step in the derivation of Eq. (2) is the calculation of $\langle V^2\rangle$. Employing Eq. (9) it suffices to calculate $\langle \dot{Q}^2\rangle$. One finds,

$$\langle E_n|\dot{Q}^2|E_n\rangle = \sum_m (E_n - E_m)^2 |\langle E_m|Q|E_n\rangle|^2. \tag{12}$$

Replacing the sum by an integral and defining

$$\hbar\omega = |E_n - E_m| \tag{13}$$

one obtains

$$\begin{aligned} \langle E_n|\dot{Q}^2|E_n\rangle = \int_0^{\infty} \hbar\omega^2 \big[&|\langle E_n + \hbar\omega | Q | E_n\rangle \rho(E + \hbar\omega) \\ &+ |\langle E_n - \hbar\omega | Q | E_n\rangle|^2 \rho(E - \hbar\omega)\big] \mathrm{d}\omega, \end{aligned} \tag{14}$$

where the two parts originate from a splitting for $E_n > E_m$ and $E_n < E_m$.

Please note, that $\hbar\omega$ in (13) is a *difference* of energies. Again, a constant shift Λ in the Hamiltonian and therefore in the energy levels does not affect the result.

Using Eq. (9) and integrating over all energy states weighted by a Boltzmann factor, one obtains

$$\begin{aligned} \langle V^2\rangle = \int_0^{\infty} |Z|^2 \hbar\omega^2 \Bigg[\int_{-\infty}^{\infty} \rho(E) f(E) & \\ \times \big[|\langle E + \hbar\omega | Q | E\rangle|^2 \rho(E + \hbar\omega) & \\ + |\langle E - \hbar\omega | Q | E\rangle|^2 \rho(E - \hbar\omega)\big] dE \Bigg] \mathrm{d}\omega. & \end{aligned} \tag{15}$$

Following [8], we denote the integrals over E in Equations (11) and (15) by

$$\begin{aligned} C_{\pm} = \int_{-\infty}^{\infty} f(E)\rho(E) \Big\{ &|\langle E + \hbar\omega | Q | E\rangle|^2 \rho(E + \hbar\omega) \\ &\pm |\langle E - \hbar\omega | Q | E\rangle|^2 \rho(E - \hbar\omega) \Big\} \mathrm{d}E \end{aligned} \tag{16}$$

One can shift $E \to E + \hbar\omega$ in the second term of $C_\pm$ yielding

$$C_\pm = \int_{-\infty}^{\infty} \rho(E)\rho(E+\hbar\omega)\,|\langle E+\hbar\omega|Q|E\rangle|^2 \times f(E)\left\{1 \pm \frac{f(E+\hbar\omega)}{f(E)}\right\}\mathrm{d}E. \quad (17)$$

As $f(E+\hbar\omega)/f(E) = \exp(-\hbar\omega/kT)$, one therefore gets

$$C_\pm = \left(1 \pm \exp\left(-\frac{\hbar\omega}{kT}\right)\right) C(\omega), \quad (18)$$

where

$$C(\omega) = \int_{-\infty}^{\infty} f(E)\rho(E)\,\rho(E+\hbar\omega)\,|\langle E+\hbar\omega|Q|E\rangle|^2\,\mathrm{d}E. \quad (19)$$

Equations (6) and (15) may therefore be written as

$$R(\omega) = \pi\omega|Z(\omega)|^2\left(1 - \exp\left(-\frac{\hbar\omega}{kT}\right)\right) C(\omega) \quad (20)$$

and

$$\langle V^2\rangle = \int_0^{\infty} \mathrm{d}\omega|Z(\omega)|^2\hbar\omega^2\left(1 + \exp\left(-\frac{\hbar\omega}{kT}\right)\right) C(\omega) \quad (21)$$

Finally, combining Eqs. (20) and (21) one arrives at (2). Hence, the derivation of the fluctuation-dissipation theorem of Callen and Welton is unaffected by a constant shift Λ of the Hamiltonian. In particular, it does not appear in Equation (3).

To conclude this section let us in addition remark that Eq. (2) is a statement about quantum mechanical and thermal fluctuations in an arbitrary system. This could, e.g. be fluctuations of a quantum harmonic oscillator in a thermal bath. These fluctuations are not necessarily in one to one correspondence to the fluctuations in fundamental quantum fields. For example, the particle in the harmonic oscillator could itself be a bound state. Moreover, the common factor Eq. (3) that is interpreted as the effect of vacuum fluctuations is essentially the average energy of one harmonic oscillator of frequency ω. However, the total vacuum energy depends on the density of states $\rho(E)$, which will be different from the true vacuum in the solid state setup of the Josephson junction[1].

IV. VACUUM FLUCTUATIONS HAVE ALREADY BEEN MEASURED BEYOND 10EV

In recent years there has been significant progress in the measurement and calculation of the Casimir force. Experiments have reached distances d below $d \sim 100\,\mathrm{nm}$ (see, e.g., [9]).

Separations of $d \sim 100\,\mathrm{nm}$ correspond to energy scales of $\frac{2\pi}{d} \sim 10\,\mathrm{eV}$. Hence, measurements of the Casimir force already probed photon zero point fluctuations with energies in excess of 10 eV. The cosmologically inferred scale of dark energy on the other hand is only of the order of $(\frac{8\pi^2 c^3}{\hbar}\rho_{\mathrm{DE}})^{\frac{1}{4}} \sim 0.01\,\mathrm{eV}$ (ρ_{DE} is the current energy density of dark energy). This is much smaller than the scales already probed by Casimir force experiments. The hypotheses that a cut off in the fluctuation spectrum might be responsible for dark energy has thus been excluded experimentally: the Casimir measurements see no such cut off.

Please note that these energies are much larger than those probed so far in the measurements of noise in Josephson junctions where frequencies up to $6 \times 10^{11}\,\mathrm{Hz}$ corresponding to $2.5 \times 10^{-3}\,\mathrm{eV}$ have been reached. When Josephson junction experiments reach $\sim 0.01\,\mathrm{eV}$, they should hence see no cut off, in agreement with Casimir force experiments.

V. CONCLUSIONS

In this brief note we tried to clarify some aspects of the question what measurements of zero point fluctuations can tell us about dark energy. We argued that in all known experiments, we only measure fluctuations of fields interacting electromagnetically. These experiments are insensitive to fluctuations of weakly interacting particles. For instance, a mechanism that adds a very weakly interacting fermion to cancel the vacuum energy contributed by the photon would not be seen in those experiments. Josephson junction experiments will not measure the absolute value of dark energy, because the noise is independent of a constant shift of the Hamiltonian. From Casimir force measurements, we know that the photon has zero point fluctuations with energies above 10 eV, much above the dark energy scale of $\sim 0.01\,\mathrm{eV}$. Hence, some strange cutoff that only affects zero point fluctuations and thereby restricts dark energy is ruled out[2].

[1] This poses a severe problem for any argument concerning dark energy in which Josephson junctions play a special role, e.g. [10].

[2] A possible loop hole is that such a cutoff affects only fluctuations that truly do not couple (aside from gravity). But those, again wouldn't be measurable.

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